Published by Institute of Physics Publishing for SISSA
Received: *March 12, 2005*
Accepted: *May 24, 2005*
Published: *June 15, 2005*

Non-SUSY p-branes delocalized in two directions, tachyon condensation and T-duality

Jian-Xin Lu

Interdisciplinary Center for Theoretical Study
University of Science and Technology of China, Hefei, Anhui 230026, China, and
Center for Mathematics and Theoretical Physics, Shanghai Institute for Advanced Study
University of Science and Technology of China, Shanghai, China, and
Interdisciplinary Center of Theoretical Studies, Chinese Academy of Sciences
Beijing 100080, China
E-mail: jxlu@ustc.edu.cn

Shibaji Roy

Saha Institute of Nuclear Physics
1/AF Bidhannagar, Calcutta-700 064, India
E-mail: roy@theory.saha.ernet.in

Abstract: We here generalize our previous construction [hep-th/0409019] of non-supersymmetric p-branes delocalized in one transverse spatial direction to two transverse spatial directions in supergravities in arbitrary dimensions (d). These solutions are characterized by five parameters. We show how these solutions in $d = 10$ interpolate between D($p+2$)-anti-D($p+2$) brane system, non-BPS D($p+1$)-branes (delocalized in one direction) and BPS Dp-branes by adjusting and scaling the parameters in suitable ways. This picture is very similar to the descent relations obtained by Sen in the open string effective description of non-BPS D($p+1$)-brane and BPS Dp-brane as the respective tachyonic kink and vortex solutions on the D($p+2$)-anti-D($p+2$) brane system (with some differences). We compare this process with the T-duality transformation which also has the effect of increasing (or decreasing) the dimensionality of the branes by one.

Keywords: p-branes, D-branes, Tachyon Condensation, String Duality.

Contents

1. Introduction

In [1] we constructed delocalized, non-supersymmetric p-brane solutions of maximal supergravities in arbitrary dimensions (d). In $d = 10$, these solutions in type-IIA (IIB) theory (for p = even (odd)) were interpreted as interpolating solutions between non-BPS D($p+1$)-branes and a codimension one BPS Dp-branes when the transverse delocalized direction was spatial. To certain extent, this picture of supergravity captures how the tachyonic kink solution on the non-BPS D($p+1$)-brane [2] from the open string effective description emerges. The picture holds even for the temporally delocalized euclidean p-brane solutions which was obtained from the spatially delocalized solution by a Wick rotation [3, 4, 1]. The emergence of this picture similar to the open string tachyon condensation might seem surprising as there is no explicit appearance of the tachyon field in the supergravity description. We, however, take it seriously and by going a step further construct the non-supersymmetric p-brane solutions delocalized in two transverse spatial directions in this paper. One of the main motivations for this construction was to see how we can understand the descent relations obtained by Sen [5, 2] in the open string effective description of BPS and non-BPS D-branes as tachyonic soliton solutions on brane-antibrane pair of higher dimension from the supergravity point of view. In fact, we will show how these solutions can be regarded as interpolating solutions between D($p+2$)-$\bar{\mathrm{D}}$($p+2$) brane systems, non-BPS D($p+1$)-branes (delocalized in one spatial transverse direction) and localized BPS Dp-branes, similar to the descent relations of Sen for the tachyonic kink (vortex) solutions on the D-$\bar{\mathrm{D}}$-brane systems [2]. We will also study the T-duality properties of these solutions.

The descent relation D($p+2$)-$\bar{\mathrm{D}}$($p+2$) $\rightarrow$ non-BPS D($p+1$) $\rightarrow$ BPS Dp obtained in the course of finding the tachyonic kink solution on the previous system involves branes on the two sides whose dimensionalities differ by two. In the absence of explicit tachyon field, the most natural way to see this picture emerging in the supergravity solution is to consider non-supersymmetric p-branes delocalized in two transverse spatial directions.

This is what we study in this paper. For the case of BPS Dp-branes the delocalized solutions [6, 7] are obtained by first periodically placing an infinite array of branes along the transverse directions (it can be done in steps when more than one directions are to be delocalized) and then taking the continuum limit.[1] This procedure assumes the 'no-force' condition among the BPS branes. However, for the non-supersymmetric branes, since they interact with each other it is not clear how this procedure will work. In [1], we obtained the delocalized, non-supersymmetric p-branes by explicitly solving the equations of motion with an appropriate metric ansatz. This gives entirely new solutions whose relation with the localized non-supersymmetric p-branes is not at all obvious unlike the BPS case.

In this paper we generalize the delocalized solutions from one transverse spatial direction to two transverse directions. The generalization is non-trivial and one needs to solve the equations of motion again to obtain these solutions. We first write the solutions in d-dimensions which are characterized by five independent parameters. We show in $d = 10$ that the solutions can be made localized $(p + 2)$-branes in two ways when the parameters satisfy certain conditions. In the first case, when there is no T-duality involved, the resulting solutions can be interpreted as D$(p+2)$-$\bar{\mathrm{D}}(p+2)$ with zero net charge and in the second case when we apply T-duality (studied in section 4) twice along the delocalized directions the resulting solutions can be interpreted as charged D$(p+2)$-$\bar{\mathrm{D}}(p+2)$ solutions. While in the first case the solutions will be characterized by two parameters, in the second case they will be characterized by three parameters. Next we will show that when the parameters satisfy some other conditions, we can convert these solutions to delocalized $(p + 1)$-branes (note that unlike in the previous case we can not make them completely localized) again in two ways. When no T-duality is involved the solutions can be interpreted as non-BPS D$(p+1)$-branes (delocalized in one direction) characterized by three parameters and when we take T-duality in one of the delocalized transverse directions, the solutions can be interpreted as charged D$(p + 1)$-$\bar{\mathrm{D}}(p + 1)$ (delocalized in one direction) solutions characterized by four parameters.[2] Note that in the latter case since we take T-duality once the theory changes from type-IIA (IIB) to IIB (IIA). Finally, we show that by appropriately scaling the parameters we can convert the solutions to localized BPS Dp-branes. So, when no T-duality is involved we interpret our solutions as interpolating solutions between D$(p+2)$-$\bar{\mathrm{D}}(p+2)$ brane system (with net charge zero), non-BPS D$(p+1)$-branes (delocalized in one direction) and BPS Dp-branes very similar to the descent relations advocated by Sen [2] in the open string effective description of non-BPS D$(p + 1)$-brane and BPS Dp-brane as the respective tachyonic kink and vortex solutions on the D$(p+2)$-anti-D$(p+2)$ brane system.

As we mentioned the process in obtaining the non-BPS D-brane (BPS D-brane) as the tachyonic kink (and vortex) solutions on the brane-antibrane systems has some similarities with the T-duality transformation. For example, for BPS D-branes, the following transition, D$(p + 2) \to$ D$(p + 1) \to$ Dp can be obtained by T-duality along the brane di-

[1]Usually this produces isometries along the transverse directions of the brane and then the application of T-duality along those directions gives the localized higher dimensional branes.

[2]We do not exactly study this case as mentioned in footnote 8. Instead we apply T-duality once to the non-susy p-brane solutions delocalized in one transverse direction [1] and obtain charged D$(p+1)$-$\bar{\mathrm{D}}(p+1)$ brane system which is fully localized (see section 4).

rections at each step. However, the crucial difference is that while for T-duality the theory changes from type-IIA (IIB) to IIB (IIA) at each step, the above process does not change the theory. We will perform T-duality on the delocalized solutions in a separate section for comparison.

This paper is organized as follows. In section 2, we write down the non-supersymmetric p-brane solutions delocalized in two transverse spatial directions in arbitrary dimensions. In section 3, we show how in $d = 10$, these solutions nicely interpolate between D$(p+2)$-$\bar{\text{D}}(p+2)$ brane system, non-BPS D$(p+1)$-branes (delocalized in one transverse direction) and the localized BPS Dp-branes similar to the descent relation of Sen. In section 4, we study the T-duality transformation of the delocalized solutions for comparison. Our conclusion is presented in section 5.

2. The delocalized solutions

In this section we give the non-supersymmetric p-brane solutions delocalized in two transverse spatial directions of maximal supergravities in arbitrary space-time dimensions d. This is a generalization of the delocalized solutions given in [1] from one transverse direction to two. The generalization is non-trivial and to obtain them one has to solve the equations of motion following from the effective action with an appropriate metric ansatz. The d-dimensional supergravity action we consider has the form,

$$S = \int d^dx\sqrt{-g}\left[R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2\cdot q!}e^{a\phi}F_{[q]}^2\right] \tag{2.1}$$

where $g_{\mu\nu}$, with $\mu,\, \nu\, =\, 0, 1, \ldots, d-1$, is the metric and $g\, =\, \det(g_{\mu\nu})$, R is the scalar curvature, ϕ is the dilaton, $F_{[q]}$ is the field strength of a $(q-1) = (d-p-3)$-form gauge field and a is the dilaton coupling. The action (2.1) represents the bosonic sector of the low energy effective action of string/M theory dimensionally reduced to d-dimensions. Now in order to obtain the delocalized solutions in two transverse directions, we have to solve the equations of motion from (2.1) with the following ansatz for the metric and the q-form field strength,

$$\begin{aligned} ds^2 &= e^{2A(r)}\left(dr^2 + r^2 d\Omega_{d-p-4}^2\right) + e^{2B(r)}\left(-dt^2 + \sum_{i=1}^{p} dx_i^2\right) + e^{2C(r)}dx_{p+1}^2 + e^{2D(r)}dx_{p+2}^2 \\ F_{[q]} &= b\ \text{Vol}(\Omega_{d-p-4}) \wedge dx_{p+1} \wedge dx_{p+2}\,. \end{aligned} \tag{2.2}$$

In the above $r = (x_{p+3}^2 + \cdots + x_{d-1}^2)^{1/2}$, $d\Omega_{d-p-4}^2$ is the line element of a unit $(d-p-4)$-dimensional sphere, $\text{Vol}(\Omega_{d-p-4})$ is its volume-form and b is the magnetic charge parameter. The solutions (2.2) represent magnetically charged p-brane solutions delocalized in transverse x_{p+1} and x_{p+2} directions. The equations of motion will be solved with the following gauge condition,

$$(p+1)B(r) + (q-3)A(r) + C(r) + D(r) = \ln G(r)\,. \tag{2.3}$$

Note that as $G(r) \to 1$, the above condition reduces to the extremality or the supersymmetry condition [8]. As mentioned in [9], the consistency of the equations of motion dictates

JHEP06(2005)026

that the non-extremality function $G(r)$ can take three different forms and we will need only one of them for our purpose which is,

$$G(r) = 1 - \frac{\omega^{2(q-3)}}{r^{2(q-3)}} = \left(1 + \frac{\omega^{q-3}}{r^{q-3}}\right)\left(1 - \frac{\omega^{q-3}}{r^{q-3}}\right) = H(r)\tilde{H}(r)\,. \tag{2.4}$$

By solving the equations of motion following from (2.1) with the ansatz (2.2) we obtain,

$$\begin{aligned}
A(r) &= -\frac{p+1}{q-1}B(r) - \ln\left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_2+\delta_3}{q-3}} + \ln\left(H\tilde{H}\right)^{\frac{1}{q-3}} \\
C(r) &= -\frac{p+1}{q-1}B(r) + \delta_2 \ln\left(\frac{H}{\tilde{H}}\right) \\
D(r) &= -\frac{p+1}{q-1}B(r) + \delta_3 \ln\left(\frac{H}{\tilde{H}}\right) \\
\phi(r) &= \frac{a(d-2)}{q-1}B(r) + \delta_1 \ln\left(\frac{H}{\tilde{H}}\right) \\
B(r) &= -\frac{2}{\chi}\ln F(r)
\end{aligned} \tag{2.5}$$

where $\chi = 2(p+1) + a^2(d-2)/(q-1)$ and the function $F(r)$ is defined as

$$F(r) = \cosh^2\theta \left(\frac{H}{\tilde{H}}\right)^{\alpha} - \sinh^2\theta \left(\frac{\tilde{H}}{H}\right)^{\beta} \tag{2.6}$$

In eqs. (2.4), (2.5), ω, δ_1, δ_2, δ_3 are real integration constants and in (2.6), θ, α, β are some real parameters. However, not all of them are independent. The parameter relations are given in the following,

$$\alpha - \beta = a\delta_1 \tag{2.7}$$

$$\frac{1}{2}\delta_1^2 + \frac{2\alpha(\alpha - a\delta_1)(d-2)}{\chi(q-1)} + \frac{2\delta_2\delta_3}{q-3} = (1 - \delta_2^2 - \delta_3^2)\frac{q-2}{q-3} \tag{2.8}$$

$$b = \sqrt{\frac{4(d-2)}{(q-1)\chi}}(q-3)\omega^{q-3}(\alpha+\beta)\sinh 2\theta\,. \tag{2.9}$$

So, from (2.7), we can determine β in terms of α and δ_1. Also, from (2.8), we can determine α in terms of δ_1, δ_2 and δ_3. Eq. (2.9) gives a relation between the charge parameter b with the other parameters. So, the solution (2.5), (2.6) contains only five independent parameters, namely, ω, θ, δ_1, δ_2 and δ_3. Writing explicitly the non-susy p-brane solutions delocalized in two transverse directions in d space-time dimensions, we have,

$$\begin{aligned}
ds^2 &= F^{\frac{4(p+1)}{(q-1)\chi}}(H\tilde{H})^{\frac{2}{q-3}}\left(\frac{H}{\tilde{H}}\right)^{-\frac{2(\delta_2+\delta_3)}{q-3}}\left(dr^2 + r^2 d\Omega^2_{d-p-4}\right) + F^{-\frac{4}{\chi}}\left(-dt^2 + \sum_{i=1}^{p} dx_i^2\right) + \\
&\quad + F^{\frac{4(p+1)}{(q-1)\chi}}\left(\frac{H}{\tilde{H}}\right)^{2\delta_2} dx^2_{p+1} + F^{\frac{4(p+1)}{(q-1)\chi}}\left(\frac{H}{\tilde{H}}\right)^{2\delta_3} dx^2_{p+2} \\
e^{2\phi} &= F^{-\frac{4a(d-2)}{(q-1)\chi}}\left(\frac{H}{\tilde{H}}\right)^{2\delta_1}, \qquad F_{[q]} = b\ \mathrm{Vol}(\Omega_{d-p-4}) \wedge dx_{p+1} \wedge dx_{p+2}\,.
\end{aligned} \tag{2.10}$$

JHEP06(2005)026

These are magnetically charged, non-supersymmetric, delocalized p-brane solutions and the corresponding electrically charged solutions can be obtained by replacing $g_{\mu\nu} \to g_{\mu\nu}$, $\phi \to -\phi$, $F \to e^{-a\phi} * F$. We note from the form of $\tilde{H}(r)$ given in (2.4) that the solutions above have potential singularities at $r = \omega$ and for avoiding this complication, we limit our discussion to the well-defined region of $r > \omega$. Also, as $r \to \infty$, H, $\tilde{H}$, $F \to 1$ and so the solutions are asymptotically flat. We like to point out that if we send H, $\tilde{H} \to 1$, such that the function $F(r)$ reduces to the usual harmonic function of a BPS p-brane then the above solutions indeed reduce to the BPS p-branes delocalized in two transverse directions. The delocalized BPS p-brane solutions can be made localized by the usual procedure of replacing the extended source by a delta function source [10]. However, this procedure does not work for the delocalized non-supersymmetric brane solutions given in (2.10). In the following sections, we will see how a fully localized $(p+2)$-brane solutions, a delocalized $(p+1)$-brane solutions as well as a fully localized p-brane solutions can be obtained from the delocalized solutions (2.10) with or without the application of T-duality. In this process we will interpret the above solutions as the interpolating solutions between D$(p+2)$-$\bar{\text{D}}(p+2)$-brane systems, non-BPS D$(p+1)$-branes delocalized in one transverse direction and BPS Dp-branes in $d = 10$.

3. Delocalized solutions as interpolating solutions

In this section we will show how the solutions given in (2.10) can be regarded as interpolating solutions between D$(p+2)$-$\bar{\text{D}}(p+2)$-brane systems, non-BPS D$(p+1)$-brane delocalized in one direction and BPS Dp-branes similar to the descent relation advocated by Sen. For this purpose we restrict our solutions in $d = 10$ and rewrite (2.10) as follows,

$$
\begin{aligned}
ds^2 &= F^{\frac{p+1}{8}} (H\tilde{H})^{\frac{2}{5-p}} \left(\frac{H}{\tilde{H}}\right)^{-\frac{2(\delta_2+\delta_3)}{5-p}} \left(dr^2 + r^2 d\Omega_{6-p}^2\right) + F^{-\frac{7-p}{8}} \left(-dt^2 + \sum_{i=1}^{p} dx_i^2\right) + \\
&\quad + F^{\frac{p+1}{8}} \left(\frac{H}{\tilde{H}}\right)^{2\delta_2} dx_{p+1}^2 + F^{\frac{p+1}{8}} \left(\frac{H}{\tilde{H}}\right)^{2\delta_3} dx_{p+2}^2 \\
e^{2\phi} &= F^{-a} \left(\frac{H}{\tilde{H}}\right)^{2\delta_1}, \qquad F_{[8-p]} = b\ \mathrm{Vol}(\Omega_{6-p}) \wedge dx_{p+1} \wedge dx_{p+2}
\end{aligned}
\tag{3.1}
$$

where $F(r)$ is as given in (2.6) with $H = 1 + \omega^{5-p}/r^{5-p}$, $\tilde{H} = 1 - \omega^{5-p}/r^{5-p}$, $a = (p-3)/2$ for Dp-branes and $a = (3-p)/2$ for NSNS branes. The parameter relations (2.7)–(2.9) take the forms,

$$
\alpha - \beta = a\delta_1 \tag{3.2}
$$

$$
\frac{1}{2}\delta_1^2 + \frac{1}{2}\alpha(\alpha - a\delta_1) + \frac{2\delta_2\delta_3}{5-p} = (1 - \delta_2^2 - \delta_3^2)\frac{6-p}{5-p} \tag{3.3}
$$

$$
b = (5-p)\omega^{5-p}(\alpha+\beta)\sinh 2\theta \tag{3.4}
$$

Using (3.2) and (3.3) we can express the parameters α, β in terms of three unknown parameters δ_1, δ_2 and δ_3 as,

$$\alpha = \pm\sqrt{2\left(1-\delta_2^2-\delta_3^2-\frac{2\delta_2\delta_3}{6-p}\right)\frac{6-p}{5-p}-\frac{(p+1)(7-p)}{16}\delta_1^2}+\frac{a\delta_1}{2}$$
$$\beta = \pm\sqrt{2\left(1-\delta_2^2-\delta_3^2-\frac{2\delta_2\delta_3}{6-p}\right)\frac{6-p}{5-p}-\frac{(p+1)(7-p)}{16}\delta_1^2}-\frac{a\delta_1}{2}\,. \tag{3.5}$$

So, the solutions (3.1) are dependent on five parameters ω, θ, δ_1, δ_2 and δ_3. Since the solutions here are non-supersymmetric, the parameters are presumably be related to the mass, charge, the tachyon vev $\langle T\rangle$ and the vev of its derivatives along the delocalized directions x_{p+1}, x_{p+2} i.e. $\langle\partial_1 T\rangle$ and $\langle\partial_2 T\rangle$ of the brane systems. However, the exact relationships between them are not clear to us. Now from (3.5) we find that the parameters δ_1, δ_2, δ_3 must satisfy the following bounds,

$$\delta_2^2+\delta_3^2+\frac{2\delta_2\delta_3}{6-p} \le 1$$
$$|\delta_1| \le 4\sqrt{\frac{2(6-p)}{(5-p)(p+1)(7-p)}}\sqrt{\left(1-\delta_2^2-\delta_3^2-\frac{2\delta_2\delta_3}{6-p}\right)}\,. \tag{3.6}$$

Once we know the form of the metric given in (3.1), we can compute the energy-momentum (e-m) tensor associated with the brane from the linearized form of the Einstein equation given by,

$$\nabla^2\left(h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h\right) = -2\kappa_0^2 T_{\mu\nu}\delta^{(7-p)}(r) \tag{3.7}$$

where we have expanded the metric around asymptotically flat space as $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$ and used the harmonic gauge $\partial_\lambda h^\lambda_\mu - \frac{1}{2}\partial_\mu h = 0$ with $h=\eta^{\mu\nu}h_{\mu\nu}$. Also in (3.7) $2\kappa_0^2 = 16\pi G_{10}$, G_{10}, being the ten dimensional Newton's constant. So, from the metric in (3.1) and (3.7) we obtain the various components of e-m tensor as,

$$\begin{aligned}
T_{00} &= \frac{\Omega_{6-p}}{2\kappa_0^2}(5-p)\omega^{5-p}\left[(\alpha+\beta)\cosh 2\theta+(\alpha-\beta)-\frac{4(\delta_2+\delta_3)}{5-p}\right]\\
T_{ij} &= -\frac{\Omega_{6-p}}{2\kappa_0^2}(5-p)\omega^{5-p}\left[(\alpha+\beta)\cosh 2\theta+(\alpha-\beta)-\frac{4(\delta_2+\delta_3)}{5-p}\right]\delta_{ij}\\
T_{p+1,p+1} &= \frac{\Omega_{6-p}}{2\kappa_0^2}(5-p)\omega^{5-p}\left[\frac{4\delta_2(6-p)+4\delta_3}{5-p}\right]\\
T_{p+2,p+2} &= \frac{\Omega_{6-p}}{2\kappa_0^2}(5-p)\omega^{5-p}\left[\frac{4\delta_3(6-p)+4\delta_2}{5-p}\right]\\
T_{mn} &= 0
\end{aligned} \tag{3.8}$$

In the above $i,j=1,\ldots,p$ and $m,n=p+3,\ldots,9$. $\Omega_n = 2\pi^{(n+1)/2}/\Gamma((n+1)/2)$ is the volume of the n-dimensional unit sphere. T_{00} in (3.8) is the mass per unit brane volume. Since it has the dimensionality of the mass per unit $(p+2)$-dimensional brane volume, we find that the energy is spread also along the delocalized directions x_{p+1} and x_{p+2}

as expected.[3] The brane is spread along x_{p+1} and x_{p+2} can also be noted from the non-vanishing components of e-m tensor $T_{p+1,p+1}$ and $T_{p+2,p+2}$. $T_{mn} = 0$ implies that the brane is localized along $x_{p+3}, \ldots, x_{d-1}$ directions and they are the true transverse directions.

Now let us look at the metrics in (3.1). They represent the metrics for non-supersymmetric p-branes delocalized in two directions x_{p+1}, x_{p+2} which are the isometric directions. For the similar case of BPS solutions they are usually made fully localized along the transverse directions by T-dualities in both x_{p+1} and x_{p+2} directions. Here we will see that the solutions in (3.1) can be made fully localized $(p+2)$-branes both with and without making T-duality transformations along x_{p+1} and x_{p+2} directions. We will consider T-duality in the next section. However, in this section no T-duality will be employed. We note from the metrics in (3.1) that they can be made localized $(p+2)$-branes if the coefficient of the term $(-dt^2 + \sum_{i=1}^{p} dx_i^2)$ match with both dx_{p+1}^2 and dx_{p+2}^2 terms. This is possible only if

$$\begin{aligned} \theta &= b = 0 \\ \delta_2 &= \delta_3 = -\frac{\alpha}{2} \end{aligned} \tag{3.9}$$

Since we have $b = 0$, the solution is chargeless and in that case we see from (2.6) that the function $F(r)$ simplifes to $F(r) = \left(\frac{H}{\tilde{H}}\right)^{\alpha}$. The form of the e-m tensor in this case can be obtained from (3.8) as,

$$\begin{aligned} T_{00} &= \frac{\Omega_{6-p}}{2\kappa_0^2}(5-p)\omega^{5-p}\left[\frac{2\alpha(7-p)}{5-p}\right] \\ T_{ij} &= -\frac{\Omega_{6-p}}{2\kappa_0^2}(5-p)\omega^{5-p}\left[\frac{2\alpha(7-p)}{5-p}\right]\delta_{ij} \\ T_{mn} &= 0 \end{aligned} \tag{3.10}$$

where $i, j = 1, 2, \ldots, (p+2)$ and $m, n = p+3, \ldots, 9$. This is exactly what we expect of localized $(p+2)$-branes. The solutions (3.1) in this case take the form

$$\begin{aligned} ds^2 &= (H\tilde{H})^{\frac{2}{5-p}}\left(\frac{H}{\tilde{H}}\right)^{\frac{p+1}{8}\alpha+\frac{2\alpha}{5-p}}\left(dr^2 + r^2 d\Omega_{6-p}^2\right) + \left(\frac{H}{\tilde{H}}\right)^{-\frac{7-p}{8}\alpha}\left(-dt^2 + \sum_{i=1}^{p+2} dx_i^2\right) \\ e^{2\phi} &= \left(\frac{H}{\tilde{H}}\right)^{-a\alpha+2\delta_1}, \qquad F_{[8-p]} = 0 \end{aligned} \tag{3.11}$$

and the parameters α and δ_1 now satisfy

$$\delta_1^2 - a\alpha\delta_1 + 2(\alpha^2 - 1)\frac{6-p}{5-p} = 0\,. \tag{3.12}$$

We identify the above solutions as D$(p+2)$-$\bar{\text{D}}(p+2)$-brane systems with zero net charge [12, 13, 9]. It should be remarked here that since $F_{[8-p]} = 0$, (3.11) can also represent non-BPS D$(p+2)$-branes [14] in the T-dual theory of the theory we start with in (2.1). With

[3] Note that the e-m tensors obtained from boundary CFT or from string field theory as given in eq. (3.51) of ref. [11] differ from those given above in the sense that the former involves a source function $f(x)$ for T_{00} and T_{ij} whereas for us they are constant. It will be interesting to find supergravity solutions which will produce such functional dependence on the components of the e-m tensor.

respect to our theory with a R-R $(8-p)$-form field strength, our above configuration should represent the chargeless D$(p+2)$-$\bar{\text{D}}(p+2)$ brane system. We also remark that this localization is possible from the delocalized solutions (3.1) without taking T-duality because of the presence of the extra parameters in the solutions. Since these extra parameters are not present in BPS solutions, localization in that case is possible only through T-duality.

Next we will try to make the p-brane solutions in (3.1) to $(p+1)$-brane solutions by equating the coefficients of the term $(-dt^2 + \sum_{i=1}^{p} dx_i^2)$ and dx_{p+1}^2. We find that this is possible if we put

$$\begin{aligned} b &= \theta = 0 \\ \delta_2 &= -\frac{\alpha}{2}, \qquad \delta_3 = \text{arbitrary}\,. \end{aligned} \tag{3.13}$$

Using (3.13) we find that the e-m tensors in (3.8) take the forms,

$$\begin{aligned} T_{00} &= \frac{\Omega_{6-p}}{2\kappa_0^2}(5-p)\omega^{5-p}\left[\frac{2\alpha(6-p)-4\delta_3}{5-p}\right] \\ T_{ij} &= -\frac{\Omega_{6-p}}{2\kappa_0^2}(5-p)\omega^{5-p}\left[\frac{2\alpha(6-p)-4\delta_3}{5-p}\right]\delta_{ij} \\ T_{p+2,p+2} &= -\frac{\Omega_{6-p}}{2\kappa_0^2}(5-p)\omega^{5-p}\left[\frac{2\alpha-4\delta_3(6-p)}{5-p}\right] \\ T_{mn} &= 0 \end{aligned} \tag{3.14}$$

where now $i,j = 1,\ldots,p+1$. It is clear from (3.14) that when the parameters are restricted as (3.13), we get $(p+1)$-brane solutions delocalized in x_{p+2}-direction. The solutions (3.1) then take the forms,

$$\begin{aligned} ds^2 &= (H\tilde{H})^{\frac{2}{5-p}}\left(\frac{H}{\tilde{H}}\right)^{\frac{p+1}{8}\alpha+\frac{\alpha-2\delta_3}{5-p}}\left(dr^2+r^2d\Omega_{6-p}^2\right) \\ &\quad + \left(\frac{H}{\tilde{H}}\right)^{-\frac{7-p}{8}\alpha}\left(-dt^2+\sum_{i=1}^{p+1}dx_i^2\right) + \left(\frac{H}{\tilde{H}}\right)^{\frac{p+1}{8}\alpha+2\delta_3} dx_{p+2}^2 \\ e^{2\phi} &= \left(\frac{H}{\tilde{H}}\right)^{-\frac{p-3}{2}\alpha+2\delta_1}, \qquad F_{[8-p]} = 0 \end{aligned} \tag{3.15}$$

and the parameters are related as,

$$\frac{1}{2}\delta_1^2 + \frac{1}{2}\alpha(\alpha - a\delta_1) - \frac{\alpha\delta_3}{5-p} = \left(1 - \frac{\alpha^2}{4} - \delta_3^2\right)\frac{6-p}{5-p}\,. \tag{3.16}$$

It can be easily checked that (3.15), (3.16) represent non-BPS D$(p+1)$-branes delocalized in x_{p+2} direction as obtained in ref. [1]. One may think that these solutions can be made localized if $T_{p+2,p+2} = 0$ or in other words if

$$\alpha = 2\delta_3(6-p)\,. \tag{3.17}$$

However, from the form of the metric in (3.15) it is clear that even in this case we do not get a localized non-BPS D$(p+1)$-brane solutions. We get misled because the e-m tensors encode only the linear properties of the metric and not the full metric. In fact it is easy to see that when (3.17) is satisfied the $\left(\frac{H}{\tilde{H}}\right)$ factor in both the terms $(dr^2 + r^2 d\Omega_{6-p}^2)$ and dx_{p+2}^2 match, but there is an additional $(H\tilde{H})^{2/(5-p)}$ factor in front of the first term which does not contribute to the linear term or the e-m tensor, but forbids the metric to take a localized non-BPS D$(p+1)$-brane form. Furthermore, we point out that even if we ignore the non-linear part of $(H\tilde{H})$ factor, the metrics in (3.1) can not be regarded as localized non-BPS D$(p+1)$ branes because the parameter relation (3.16) differs from that of a localized non-BPS brane solutions [9].

Now we will see how the delocalized solutions in (3.1) reduce to BPS p-branes. The necessary condition for this to happen is that the e-m tensor in (3.8) must satisfy $T_{00} = -T_{ii}$, for $i = 1, \ldots, p$ and $T_{mm} = 0$ for $m = p+1, \ldots, 9$. From (3.8) we find that this condition implies that either $\delta_2, \delta_3 \to 0$, or, $\omega^{5-p} \to 0$, $\theta \to \infty$ such that $\omega^{5-p}\cosh 2\theta =$ finite. Examining the metric in (3.1) carefully, we find that we have the correct BPS limits if we scale the parameters as

$$\begin{aligned} \omega^{5-p} &\to \epsilon\bar{\omega}^{5-p} \\ (\alpha+\beta)\sinh 2\theta &\to \epsilon^{-1} \end{aligned} \tag{3.18}$$

where $\epsilon \to 0$ is a dimensionless parameter. Also from (3.4), (2.6) we find that under the above scaling $b \to (5-p)\bar{\omega}^{5-p}$, $F \to \bar{H} = 1 + \bar{\omega}^{5-p}/r^{5-p}$ and H, $\tilde{H} \to 1$. Now since δ's are bounded given by (3.6), it can be easily checked that the configurations (3.1) reduce to

$$\begin{aligned} ds^2 &= \bar{H}^{\frac{p+1}{8}}\left(dr^2 + r^2 d\Omega_{6-p}^2 + dx_{p+1}^2 + dx_{p+2}^2\right) + \bar{H}^{-\frac{7-p}{8}}\left(-dt^2 + \sum_{i=1}^{p} dx_i^2\right) \\ e^{2\phi} &= \bar{H}^{-a}, \qquad F_{[8-p]} = b\mathrm{Vol}(\Omega_{6-p}) \wedge dx_{p+1} \wedge dx_{p+2}\,. \end{aligned} \tag{3.19}$$

This is the BPS Dp-brane solutions delocalized in x_{p+1} and x_{p+2} directions. The components of e-m tensor in (3.8) take the forms, $T_{00} = -T_{ii} = \frac{\Omega_{6-p}}{2\kappa_0^2}(5-p)\bar{\omega}^{5-p}$ for $i = 1, \ldots, p$ and $T_{p+1,p+1} = 0$, $T_{p+2,p+2} = 0$, $T_{mm} = 0$. Although the BPS configuration we get in (3.19) is delocalized, this delocalization is trivial as opposed to the delocalized solutions we got for non-BPS D$(p+1)$-branes in (3.15). This is because we can localize the above solutions by replacing the membrane-like source along x_{p+1}, x_{p+2} by a point source or delta function source without any cost of energy (true for BPS branes). In calculating the e-m tensor we replace the Poisson's equation of the harmonic function $\bar{H}(r)$ as [10]

$$\nabla^2\bar{H} = -\Omega_{6-p}(5-p)\bar{\omega}^{5-p}\delta^{(7-p)}(r) \quad \Rightarrow \quad \nabla^2\bar{H} = -\Omega_{8-p}(7-p)\bar{\omega}^{7-p}\delta^{(9-p)}(r)\,. \tag{3.20}$$

The harmonic function now takes the form $\bar{H}(r) = 1 + \bar{\omega}^{7-p}/r^{7-p}$, where r includes x_{p+1} and x_{p+2}. The components of e-m tensors will be given as $T_{00} = -T_{ii} = \frac{\Omega_{8-p}}{2\kappa_0^2}(7-p)\bar{\omega}^{7-p}$, for, $i = 1, \ldots, p$ and $T_{mm} = 0$, for, $m = p+1, \ldots, 9$ and the configuration (3.19) will reduce to the localized BPS Dp-brane solutions.

This therefore shows how the non-supersymmetric p-brane solutions delocalized in two transverse spatial directions eq. (3.1) can be interpreted as the interpolating solutions between D$(p+2)$-$\bar{\text{D}}(p+2)$ systems (eqs. (3.11), (3.12)), non-BPS D$(p+1)$-branes delocalized in one direction (eqs. (3.15), (3.16)) and localized BPS Dp-branes (eq. (3.19)). This picture is very similar to the descent relations obtained by Sen for the tachyonic kink and vortex solutions on the brane-antibrane systems [2]. However, there are some differences, particularly, for the intermediate state i.e. the non-BPS D$(p+1)$-branes starting from the brane-antibrane systems. For the case of Sen's descent relation the non-BPS D$(p+1)$-brane obtained as a kink solution interpolating tachyon vacua is a localized one, whereas, we obtain the non-BPS D$(p+1)$-branes delocalized in one direction which, unlike the BPS branes, we do not know how to localize.

4. The delocalized solutions and T-duality

In this section we will study the T-duality properties of the delocalized solutions given in section 2. Since the solutions obtained in section 2, have the interpretation of interpolating solutions of brane configurations whose dimensionalities differ by one, similar to the T-duality transformations, we study this property for comparison of the results obtained in section 3. But before we study the T-duality of the non-supersymmetric p-brane solutions delocalized in two transverse directions, we study the T-duality for the solutions delocalized in one transverse direction obtained in ref. [1].

Let us write down here the non-supersymmetric p-brane solutions delocalized in one transverse spatial direction in $d = 10$,

$$\begin{aligned} ds^2 &= F^{\frac{p+1}{8}}(H\tilde{H})^{\frac{2}{6-p}}\left(\frac{H}{\tilde{H}}\right)^{-\frac{2\delta_2}{6-p}}\left(dr^2 + r^2 d\Omega_{7-p}^2\right) + F^{-\frac{7-p}{8}}\left(-dt^2 + \sum_{i=1}^{p} dx_i^2\right) + \\ &\quad + F^{\frac{p+1}{8}}\left(\frac{H}{\tilde{H}}\right)^{2\delta_2} dx_{p+1}^2 \\ e^{2\phi} &= F^{-\frac{p-3}{2}}\left(\frac{H}{\tilde{H}}\right)^{2\delta_1}, \qquad F_{[8-p]} = b\ \mathrm{Vol}(\Omega_{7-p}) \wedge dx_{p+1} \end{aligned} \tag{4.1}$$

where the function $F(r)$ is as given in (2.6). One can localize the above solutions along x_{p+1} direction without taking T-duality when the parameters satisfy certain condition. The resulting solution can be identified with non-BPS D$(p+1)$-brane which is chargeless and was studied in ref. [1]. Here we will localize the above solutions by applying T-duality along x_{p+1}-direction. For that purpose we first write the metric in (4.1) in the string frame as,

$$\begin{aligned} ds^2_{\mathrm{str.}} &= e^{\phi/2} ds^2 \\ &= F^{\frac{1}{2}}(H\tilde{H})^{\frac{2}{6-p}}\left(\frac{H}{\tilde{H}}\right)^{-\frac{2\delta_2}{6-p}+\frac{\delta_1}{2}}\left(dr^2 + r^2 d\Omega_{7-p}^2\right) + \\ &\quad + F^{-\frac{1}{2}}\left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{2}}\left(-dt^2 + \sum_{i=1}^{p} dx_i^2\right) + F^{\frac{1}{2}}\left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{2}+2\delta_2} dx_{p+1}^2\,. \end{aligned} \tag{4.2}$$

After making a T-duality transformation [10, 15, 16] the $(p+1, p+1)$ component of the string frame metric in the dual theory will be given as,

$$\tilde{g}^{\mathrm{str.}}_{p+1,p+1} = F^{-\frac{1}{2}}\left(\frac{H}{\tilde{H}}\right)^{-\frac{\delta_1}{2}-2\delta_2}. \tag{4.3}$$

The rest of the metric components remain unaltered. The dilaton in the dual theory takes the form

$$e^{2\tilde{\phi}} = F^{\frac{2-p}{2}} \left(\frac{H}{\tilde{H}}\right)^{\frac{3\delta_1}{2} - 2\delta_2} . \tag{4.4}$$

We now rewrite the dual frame metric from the string frame to the Einstein frame as,

$$\begin{aligned} d\tilde{s}^2 &= e^{-\tilde{\phi}/2} d\tilde{s}^2_{\mathrm{str.}} \\ &= F^{\frac{p+2}{8}} (H\tilde{H})^{\frac{2}{6-p}} \left(\frac{H}{\tilde{H}}\right)^{-\frac{2\delta_2}{6-p} + \frac{\delta_2}{2} + \frac{\delta_1}{8}} \left(dr^2 + r^2 d\Omega^2_{7-p}\right) + \\ &\quad + F^{-\frac{6-p}{8}} \left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_2}{2} + \frac{\delta_1}{8}} \left(-dt^2 + \sum_{i=1}^{p} dx_i^2\right) + F^{-\frac{6-p}{8}} \left(\frac{H}{\tilde{H}}\right)^{-\frac{7\delta_1}{8} - \frac{3\delta_2}{2}} dx^2_{p+1} . \end{aligned} \tag{4.5}$$

Note that in the above the dual frame metric $d\tilde{s}^2_{\mathrm{str.}}$ has the same form as in (4.2) with the $(p+1, p+1)$ component as given in (4.3).[4] We observe that (4.5) can indeed be made localized $(p+1)$-brane if we put[5]

$$\delta_1 = -2\delta_2 . \tag{4.6}$$

The complete solutions then take the forms,

$$\begin{aligned} d\tilde{s}^2 &= F^{\frac{p+2}{8}} (H\tilde{H})^{\frac{2}{6-p}} \left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{6-p} - \frac{\delta_1}{8}} \left(dr^2 + r^2 d\Omega^2_{7-p}\right) + F^{-\frac{6-p}{8}} \left(\frac{H}{\tilde{H}}\right)^{-\frac{\delta_1}{8}} \left(-dt^2 + \sum_{i=1}^{p+1} dx_i^2\right) \\ e^{2\tilde{\phi}} &= F^{\frac{2-p}{2}} \left(\frac{H}{\tilde{H}}\right)^{\frac{5\delta_1}{2}} , \qquad F_{[7-p]} = b \mathrm{Vol}(\Omega_{7-p}) . \end{aligned} \tag{4.7}$$

The parameters are related as,

$$\alpha - \beta = a\delta_1 \tag{4.8}$$

$$b = (6-p)(\alpha + \beta)\omega^{6-p} \sinh 2\theta \tag{4.9}$$

$$\frac{1}{2}\delta_1^2 + \frac{\delta_1^2}{4}\frac{7-p}{6-p} + \frac{\alpha(\alpha - a\delta_1)}{2} = \frac{7-p}{6-p} . \tag{4.10}$$

So, there are only three independent parameters characterizing the solutions namely, δ_1, ω and θ. We point out the unlike the localization obtained in ref. [1], the localization obtained here by T-duality give charged $(p+1)$-brane solutions. Also if the original delocalized p-brane solutions belong to type-IIA (IIB) theory, then the localized solutions (4.7) obtained

[4]From the magnetic charge as given in (4.7), this configuration appears to represent the charged D$(p+1)$-$\bar{\mathrm{D}}(p+1)$ brane system. However, from the following localization process, we see that this configuration contains more than the above system. Examining the metric in (4.5) and our experience in the brane bound state, we conclude that the above configuration actually represents the charged D$(p+1)$-$\bar{\mathrm{D}}(p+1)$ system with non-BPS Dp brane uniformly distributed along x_{p+1} direction. In other words, the above configuration represents a bound state of charged (D$(p+1)$,$\bar{\mathrm{D}}(p+1)$) system and non-BPS Dp branes. The following localization process removes the delocalized non-BPS Dp brane along the x_{p+1} direction.

[5]We remark that all the δ's here need not be negative as otherwise stated in our previous paper [1]. We noticed this after that paper was published and this, however, does not change any of the conclusions of the paper.

by T-duality belong to type-IIB (IIA) theory. We identify the solutions (4.7) in the dual theory as the charged D($p+1$)-$\bar{\rm D}$($p+1$) brane systems [9]. Let us recall that the chargeless solutions obtained in [1] without T-duality were identified as non-BPS D($p+1$)-branes. Comparing the solutions (4.7) with the non-supersymmetric, charged (p+1)-brane solutions obtained in eq. (2.30) of ref. [9] in $d=10$, we find that they match exactly if we identify,

$$\begin{aligned}
\hat{F} &= F\left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{6-p}} \\
\hat{\delta} &= \frac{7-p}{6-p}\delta_1 \\
\hat{\alpha} &= \alpha + \frac{\delta_1}{6-p}
\end{aligned} \tag{4.11}$$

where we have denoted the function F and the parameters in the solutions of ref. [9] with a 'hat' to avoid any confusion. We can easily check that the parameter relation (4.10) indeed match with the relation (2.26) of ref. [9] in $d=10$ under the above identifications. We have thus shown that starting from the delocalized (in one transverse direction) non-supersymmetric Dp-brane solutions we can obtain a localized, charged D($p+1$)-$\bar{\rm D}$($p+1$)-brane systems provided the parameters in the original solutions satisfy eqs. (4.6). Conversely, one can also start from charged D($p+1$)-$\bar{\rm D}$($p+1$)-brane systems and applying T-duality along x_{p+1}-direction, obtain the delocalized non-supersymmetric Dp-branes given in ref. [1] if the parameters are identified as in (4.11) and satisfy (4.6).

Having described the T-duality properties of the non-supersymmetric Dp branes delocalized in one transverse direction, we proceed to study the T-duality of the delocalized solutions in two transverse directions given in (3.1). In the previous section we saw how the delocalized solutions can be made localized without taking T-duality and the resulting solutions were identified as D(p+2)-$\bar{\rm D}$(p+2)-brane systems with zero net charge. However, we know that the same theory also contains D(p+2)-$\bar{\rm D}$(p+2)-brane solutions with non-zero RR charges. In this section we will show that the solutions (3.1) can also be localized by taking T-duality twice along the delocalized directions x_{p+1} and x_{p+2} and this procedure will produce the charged D(p+2)-$\bar{\rm D}$(p+2)-brane systems that we just mentioned. In order to apply T-duality let us rewrite the solutions (3.1) in the string frame as,

$$\begin{aligned}
ds^2_{\rm str.} &= e^{\phi/2}ds^2 \\
&= F^{\frac{1}{2}}(H\tilde{H})^{\frac{2}{5-p}}\left(\frac{H}{\tilde{H}}\right)^{-\frac{2(\delta_2+\delta_3)}{5-p}+\frac{\delta_1}{2}}\left(dr^2+r^2d\Omega^2_{6-p}\right)+ \\
&\quad +F^{-\frac{1}{2}}\left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{2}}\left(-dt^2+\sum_{i=1}^{p}dx_i^2\right)+F^{\frac{1}{2}}\left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{2}+2\delta_2}dx^2_{p+1}+ \\
&\quad +F^{\frac{1}{2}}\left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{2}+2\delta_3}dx^2_{p+2} \\
e^{2\phi} &= F^{\frac{3-p}{2}}\left(\frac{H}{\tilde{H}}\right)^{2\delta_1}, \qquad F_{[8-p]} = b{\rm Vol}(\Omega_{6-p})\wedge dx_{p+1}\wedge dx_{p+2}\,.
\end{aligned} \tag{4.12}$$

Applying T-duality[6] [10, 15, 16] along x_{p+2} and x_{p+1}, we obtain the $(p+2,p+2)$ and $(p+1,p+1)$ components of the string frame metric in the dual theory as,

$$\begin{aligned}\tilde{g}^{\rm str.}_{p+2,p+2} &= F^{-\frac{1}{2}}\left(\frac{H}{\tilde{H}}\right)^{-\frac{\delta_1}{2}-2\delta_3}\\ \tilde{g}^{\rm str.}_{p+1,p+1} &= F^{-\frac{1}{2}}\left(\frac{H}{\tilde{H}}\right)^{-\frac{\delta_1}{2}-2\delta_2}\,.\end{aligned} \tag{4.13}$$

The rest of the metric components remain the same. The dilaton in the dual theory takes the form,

$$e^{2\tilde{\phi}} = F^{-\frac{p-1}{2}}\left(\frac{H}{\tilde{H}}\right)^{\delta_1-2\delta_2-2\delta_3}\,. \tag{4.14}$$

Now we rewrite the T-dual solutions in the same theory with the metric in the Einstein frame using (4.12)–(4.14) as,[7]

$$\begin{aligned}d\tilde{s}^2 &= e^{-\tilde{\phi}/2}d\tilde{s}^2_{\rm str.}\\ &= F^{\frac{p+3}{8}}(H\tilde{H})^{\frac{2}{5-p}}\left(\frac{H}{\tilde{H}}\right)^{-\frac{2(\delta_2+\delta_3)}{5-p}+\frac{\delta_1}{4}+\frac{(\delta_2+\delta_3)}{2}}\left(dr^2+r^2d\Omega^2_{6-p}\right)+\\ &\quad +F^{-\frac{5-p}{8}}\left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{4}+\frac{(\delta_2+\delta_3)}{2}}\left(-dt^2+\sum_{i=1}^{p}dx_i^2\right)+F^{-\frac{5-p}{8}}\left(\frac{H}{\tilde{H}}\right)^{-\frac{3\delta_1}{4}-\frac{3\delta_2}{2}+\frac{\delta_3}{2}}dx^2_{p+1}+\\ &\quad +F^{-\frac{5-p}{8}}\left(\frac{H}{\tilde{H}}\right)^{-\frac{3\delta_1}{4}+\frac{\delta_2}{2}-\frac{3\delta_3}{2}}dx^2_{p+2}\\ e^{2\tilde{\phi}} &= F^{-\frac{p-1}{2}}\left(\frac{H}{\tilde{H}}\right)^{\delta_1-2\delta_2-2\delta_3}\,,\qquad F_{[6-p]}=b{\rm Vol}(\Omega_{6-p})\,.\end{aligned} \tag{4.15}$$

Thus we note from above that the metric can be localized if the coefficients of $(-dt^2+\sum_{i=1}^{p}dx_i^2)$, dx^2_{p+1} and dx^2_{p+2} match and this happens if the parameters satisfy,

$$\delta_1 = -2\delta_2 = -2\delta_3 \tag{4.16}$$

So, the localized solutions have only three independent parameters, namely, ω, θ and δ_1. The solutions then take the forms,

$$\begin{aligned}d\tilde{s}^2 &= F^{\frac{p+3}{8}}(H\tilde{H})^{\frac{2}{5-p}}\left(\frac{H}{\tilde{H}}\right)^{\frac{2\delta_1}{5-p}-\frac{\delta_1}{4}}\left(dr^2+r^2d\Omega^2_{6-p}\right)+F^{-\frac{5-p}{8}}\left(\frac{H}{\tilde{H}}\right)^{-\frac{\delta_1}{4}}\left(-dt^2+\sum_{i=1}^{p+2}dx_i^2\right)\\ e^{2\tilde{\phi}} &= F^{-\frac{p-1}{2}}\left(\frac{H}{\tilde{H}}\right)^{3\delta_1}\,,\qquad F_{[6-p]}=b{\rm Vol}(\Omega_{6-p})\end{aligned} \tag{4.17}$$

[6] Although we do not explicitly write the configuration by taking T-duality once along x_{p+1} in (4.12), but, it can be easily shown that applying T-duality in that case will lead to charged D$(p+1)$-$\bar{\rm D}(p+1)$ brane system delocalized in x_{p+2} direction and characterized by four parameters.

[7] By the same token as given in the footnote 6, the following configuration actually represents a rather complicated bound state of charged (D$(p+2)$, $\bar{\rm D}(p+2)$) system, non-BPS D$(p+1)_1$, non-BPS D$(p+1)_2$ and chargeless (Dp, $\bar{\rm D}p$) system where the subscripts '1' and '2' refer to the non-BPS D$(p+1)$ branes with one of their directions being along x_{p+1} and x_{p+2} respectively. The following localization process removes all the branes but the the charged D$(p+2)$-$\bar{\rm D}(p+2)$ brane system.

where the parameters satisfy the relations,

$$
\begin{aligned}
&\alpha - \beta = a\delta_1 \\
&b = (5-p)\omega^{5-p}(\alpha+\beta)\sinh 2\theta \\
&\frac{1}{2}\delta_1^2 + \frac{1}{2}\alpha(\alpha - a\delta_1) + \frac{\delta_1^2}{4(5-p)} = \left(1 - \frac{\delta_1^2}{2}\right)\frac{6-p}{5-p}\,.
\end{aligned}
\tag{4.18}
$$

We identify the above solutions as the charged D$(p+2)$-$\bar{\mathrm{D}}(p+2)$-brane [9] systems. Recall that the localized solutions obtained in section 3 without taking T-duality were chargeless and the corresponding solutions were identified as chargeless D$(p+2)$-$\bar{\mathrm{D}}(p+2)$-brane systems. Unlike the case of solutions with one delocalized direction, the localized solutions obtained here with or without T-duality belong to the same theory i.e. either to type-IIA (or IIB) theory because we have taken T-duality twice. Indeed comparing the non-supersymmetric, localized charged D$(p+2)$-$\bar{\mathrm{D}}(p+2)$-brane solutions given in eq.(2.30) of ref. [9] in $d=10$, we find that they match with (4.15) provided,

$$
\begin{aligned}
\hat{F} &= F\left(\frac{H}{\tilde{H}}\right)^{\frac{2\delta_1}{5-p}} \\
\hat{\delta} &= \frac{2(7-p)}{(5-p)}\delta_1 \\
\hat{\alpha} &= \alpha + \frac{2\delta_1}{5-p}
\end{aligned}
\tag{4.19}
$$

where we have denoted the quantities in the solutions (2.30) of ref. [9] with a 'hat'. We can easily check that the parameter relation given by the last equation of (4.18) goes over to the parameter relation (2.26) of ref. [9] in $d=10$ under the above identification. This concludes our discussion on T-duality. The main difference between the localization obtained without T-duality and with T-duality is that in the former case the solutions are chargeless whereas, for the latter case the solutions are charged.

5. Conclusion

To summarize, we have obtained in this paper the non-supersymmetric p-brane solutions delocalized in two transverse spatial directions which arise as solutions of d-dimensional supergravity containing a metric, a dilaton and a $(d-p-3)$-form gauge field. The solutions are characterized by five parameters. By adjusting and scaling the parameters we have shown how these solutions in $d=10$ can be interpreted as interpolating solutions between D$(p+2)$-$\bar{\mathrm{D}}(p+2)$-brane systems, non-BPS D$(p+1)$-branes (delocalized in one direction) and localized BPS Dp-branes. This picture is very similar to the descent relations proposed by Sen for the tachyonic kink and vortex solutions on the brane-antibrane system. For the case of descent relations all the brane configurations are localized i.e. as the tachyon condenses, the energy of the system can be shown to get localized to a codimension one brane at each step. So, starting from a brane-antibrane system of dimensionality $(p+2)$, one first gets a localized non-BPS brane of dimensionality $(p+1)$ and then gets a localized BPS brane of dimensionality p. However, in our case we do not get a localized non-BPS D$(p+1)$-brane in the intermediate step. The reason is that for the non-supersymmetric branes we do not know the relation between the localized solution and the delocalized solution unlike the case of BPS branes. The interpretation of the delocalized solutions as the interpolating solutions and their similarity with the tachyonic solitons obtained by Sen suggests that the dynamics of tachyon condensation is perhaps related to the movements in the parameter space associated with the solutions. However, to make this statement more concrete it is necessary to give a microscopic string interpretation of the solutions found in this paper and relate the parameters with the physical parameters of the brane system. For the case of localized solutions one such possible interpretation was given in ref. [12, 14] and it will be interesting to find similar relations for the delocalized solutions of this paper as well.

Since in interpreting the delocalized solutions as interpolating solutions we have not used T-duality which also has the effect of localizing and changing the dimensionality of the brane solutions by one, we have studied this aspect in a separate section for comparison. We have studied the T-duality of the solutions delocalized in both one and two transverse directions. For the former case T-duality produces a localized D$(p+1)$-$\bar{\text{D}}(p+1)$ brane system which is charged and changes the theory from type-IIA (IIB) to IIB (IIA). For the latter case T-duality produces D$(p+2)$-$\bar{\text{D}}(p+2)$-brane system which is charged as opposed to the chargeless solution one gets without T-duality. But in both cases (with or without T-duality) the solutions belong to the same theory.

Acknowledgments

JXL would like to thank the Michigan Center for Theoretical Physics for hospitality during the final stage of this work. He also acknowledges support by grants from the Chinese Academy of Sciences and the grant from the NSF of China with Grant No: 90303002.

References

[1] J.X. Lu and S. Roy, *Delocalized, non-susy p-branes, tachyon condensation and tachyon matter*, *JHEP* **11** (2004) 008 [hep-th/0409019].

[2] A. Sen, *Non-BPS states and branes in string theory*, hep-th/9904207.

[3] A. Sen, *Rolling tachyon*, *JHEP* **04** (2002) 048 [hep-th/0203211]; *Tachyon matter*, *JHEP* **07** (2002) 065 [hep-th/0203265]; *Field theory of tachyon matter*, *Mod. Phys. Lett.* **A 17** (2002) 1797 [hep-th/0204143].

[4] K. Ohta and T. Yokono, *Gravitational approach to tachyon matter*, *Phys. Rev.* **D 66** (2002) 125009 [hep-th/0207004].

[5] A. Sen, *Descent relations among bosonic D-branes*, *Int. J. Mod. Phys.* **A 14** (1999) 4061 [hep-th/9902105].

[6] H. Lu, C.N. Pope and K.S. Stelle, *Vertical versus diagonal dimensional reduction for p-branes*, *Nucl. Phys.* **B 481** (1996) 313 [hep-th/9605082].

[7] R.R. Khuri and R.C. Myers, *Rusty scatter branes*, *Nucl. Phys.* **B 466** (1996) 60 [hep-th/9512061].

[8] M.J. Duff, R.R. Khuri and J.X. Lu, *String solitons*, *Phys. Rept.* **259** (1995) 213 [hep-th/9412184].

[9] J.X. Lu and S. Roy, *Static, non-susy p-branes in diverse dimensions*, *JHEP* **02** (2005) 001 [hep-th/0408242].

[10] J.C. Breckenridge, G. Michaud and R.C. Myers, *More D-brane bound states*, *Phys. Rev.* **D 55** (1997) 6438 [hep-th/9611174].

[11] A. Sen, *Tachyon dynamics in open string theory*, hep-th/0410103.

[12] J.X. Lu and S. Roy, *Supergravity approach to tachyon condensation on the brane- antibrane system*, *Phys. Lett.* **B 599** (2004) 313 [hep-th/0403147].

[13] M. Bertolini et al., *Is a classical description of stable non-BPS D-branes possible?*, *Nucl. Phys.* **B 590** (2000) 471 [hep-th/0007097].

[14] P. Brax, G. Mandal and Y. Oz, *Supergravity description of non-BPS branes*, *Phys. Rev.* **D 63** (2001) 064008 [hep-th/0005242].

[15] S.V. Klimenko, I.N. Nikitin and V.V. Talanov, *Singularities on the world sheets of open relativistic strings*, hep-th/9505081.

[16] A. Das and S. Roy, *On M-theory and the symmetries of type-II string effective actions*, *Nucl. Phys.* **B 482** (1996) 119 [hep-th/9605073].

JHEP06(2005)026